COMPUTATIONAL GAME THEORY

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Modern Business is not just…

- Outsmarting competition
- Grabbing market share
- Fighting back
- Beating opponents
- Locking customers

** This is Poor Strategy!

The Better Strategy

- Listen to customers;
- Work with suppliers as strategic partners and even with competitors.
- With lack of cooperation you can lose all you made so far.

An Example . . .

- U.S. Airlines lost all the profits made until 1990 during the period 1990-1993 in a tough price war.
- Today we are watching as the U.S. dollar loses value with endless wars.

You succeed only when others succeed:

- Hindu scriptures say: ***Sarveh Janaaste sukhinobhavantu-Samasta san mangalani santu***
- **In other words … Let all people live in happiness. Let there be peace all around.**

What is business then?

- It is just like running a good family.
- Parents and children **cooperate** to create the best pie. They fight and **compete** to share the fruits of cooperation.
- In a sense modern game theory can as well be dubbed **coopetition**.

Players in business are both Mr Jekyll and Mr Hyde.

Konjam nilavu, konjam neruppu.

Konjam mirugam, konjam kadavul

(Tamil song)

- In other words . . . sometimes she is cool as a moon, sometimes ferocious as fire, sometimes she is an animal, at other times an angel.

Where is Game Theory Applied?

- Winning elections
- Resolving legal disputes
- Apportioning environmental costs
- Measuring the importance of factors that contribute to the spread of diseases
- Choosing mates

The list is endless.

Nim Games

A Win - Lose Game

69 54 18 46

Remove some (at least 1) from a non empty basket.
The player who removes the last one wins.

Who will win?

Ann. Math. 1903 (Bouton)

Key idea

No matter what you do, the opponent gets one more chance to move and leave the situation the same way.

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	0	0	0	1	0	1
	1	1	0	1	1	0
	0	1	0	0	1	0
	1	0	1	1	1	0
1	2	2	1	3	3	1

You can remove from basket 1 some amount that will leave all column sums even.

Existence vs. Winning Strategy

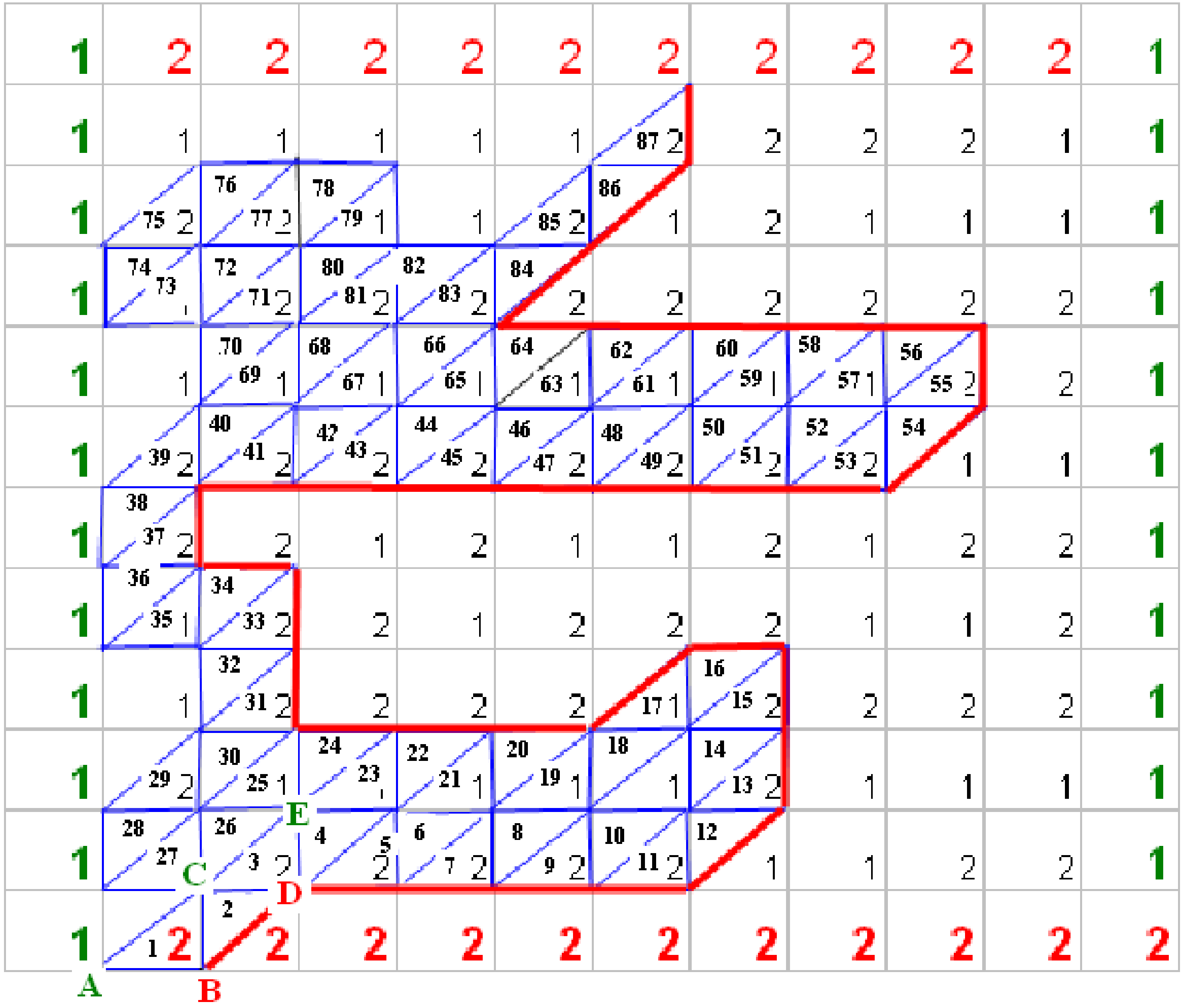
- Perfect information games
- Zermelo’s theorem
- A combinatorial game of perfect information with an algorithm to locate the winner
- Game of Hex

Hex Board

1	2	2	2	2	2	2	2	2	2	2	1	
1	1	1	1	1	1	2	2	2	2	1	1	
1	2	2	1	1	2	1	2	1	1	1	1	
1	1	2	2	2	2	2	2	2	1	2	1	
1	1	2	1	1	1	1	1	1	2	2	1	
1	2	2	2	2	2	2	2	2	1	1	1	
1	2	2	1	2	1	1	2	1	2	2	1	
1	1	2	2	1	2	2	2	1	1	2	1	
1	1	2	2	2	2	1	2	2	2	2	1	
1	2	1	1	1	1	1	2	1	1	1	1	
1	2	2	2	2	2	2	1	1	2	2	1	
1	2	2	2	2	2	2	2	2	2	2	2	

A 10 x 10 Hex Board with Frame

Game of Hex



2-3 Finger Morra

- **Let row player choose rows 1, 2 with probability x, 1-x and column player choose columns 1, 2 with chance y, -y**

	4	-5
	-5	6

Minimax Theorem

Theorem 1.1 (Minimax Theorem). *Given a payoff matrix* $A = (a_{ij})_{m\times n}$, *there exists a pair of probability distributions* $\mathbf{p} = (p_1, \ldots, p_m)$ *on the rows of* A *and* $\mathbf{q} = (q_1, \ldots, q_n)$ *on the columns of* A *such that*

$$\sum_i a_{ij} p_i \geq v \quad \forall\, j = 1 \ldots, n,$$

and

$$\sum_j a_{ij} q_j \leq v \quad \forall\, i = 1 \ldots, m$$

for a unique constant v

A guessing game by II of I’s choice

$$\begin{array}{c} & \begin{array}{ccccc} (1,2) & (1,3) & 2 & (3,1) & (3,2) \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{pmatrix} 1 & 1 & 2 & 2 & 3 \\ 2 & 3 & 1 & 3 & 2 \\ 3 & 2 & 2 & 1 & 1 \end{pmatrix} \end{array}$$

The problem as linear inequalities

When I uses row 1,

$$4q_1 + 3q_2 + 2(1 - 2q_1 - 2q_2) \leq v$$

When I uses row 2,

$$4q_1 + 6q_2 + (1 - 2q_1 - 2q_2) \leq v$$

When I uses row 3,

$$4q_1 + 3q_2 + 2(1 - 2q_1 - 2q_2) \leq v$$

Solution to the example

When II uses column 1

$$p_1 + 2p_2 + 3p_3 = 4p_1 + 2(1 - 2p_1) \geq v$$

When II uses column 2,

$$p_1 + 3p_2 + 2p_3 = 3p_1 + 3(1 - 2p_1) = 3 - 3p_1 \geq v.$$

II chooses column 3

$$2p_1 + p_2 + 2p_3 \geq v$$

Thus $v \leq \frac{9}{5}$.

Solution to the example

When II uses column 1

$$p_1 + 2p_2 + 3p_3 = 4p_1 + 2(1 - 2p_1) \geq v$$

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$$p_1 + 3p_2 + 2p_3 = 3p_1 + 3(1 - 2p_1) = 3 - 3p_1 \geq v.$$

II chooses column 3

$$2p_1 + p_2 + 2p_3 \geq v$$

Thus $v \leq \frac{9}{5}$.

Optimal strategies for the game

Thus a good strategy for Player II is

avoid choices $(1, 2) and (3, 2)$

use the choice 2 with chance $\frac{3}{5}$.

use choices $(1, 3)$ or $(3, 1)$ with chance $\frac{1}{5}$

$p_1 = p_3 = \frac{2}{5}$ and $p_2 = \frac{1}{5}$.

Exercise: Solve the game

$$\begin{bmatrix} 2 & 5 \\ 5 & 0 \\ 9 & -4 \\ -2 & 12 \end{bmatrix}$$

Solution to the exercise

$$\max\left\{\frac{25}{8}, \frac{53}{16}, 2, 0, \frac{60}{19}, \frac{100}{27}\right\} = \frac{100}{27}$$

$$\mathbf{p} = \left(0, 0, \frac{14}{27}, \frac{13}{27}\right)^T, and \quad \mathbf{q} = \left(\frac{16}{27}, \frac{11}{27}\right).$$

Nash Equilibrium- a motivation

Example 5 A monopolist farmer wants to sell pumpkins at his village farm market for the upcoming Halloween festival. He has harvested 1000 pumpkins. The price per pumpkin(in cents) is a function of his supply given by

$$\begin{aligned} p &= 600 - s \quad \textit{if } s < 600 \\ &= 0 \qquad\qquad \textit{if } \ s \geq 600 \end{aligned}$$

The farmer would like to maximize his profit by controlling the supply s of pumpkins to the market. Now

$$\max_{0 \leq s \leq 1000} s \cdot \max(600 - s, 0) = \max_{0 \leq s \leq 1000} s\,(600 - s)$$

which gives the maximum at $s^* = 300$.

With two farmers and one cooperative

Suppose farmer I brings 150

Farmer II will then maximize his gross income

$$\max s_2 (450 - s_2) \text{ and thus his optimal supply will be } \quad s_2^* = 225.$$

Of course, the betrayal from any cooperative commitment is noticed by player I

Suppose I brings 200 pumpkins

$s_2^* = 200$ pumpkins.