Complex numbers

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We will do the following 2 problems on complex nos.

(1) Let $z \in \mathbb{C}$. Then $z > \Leftrightarrow Re(z^n) > 0 \quad \forall n$.

(2) Let $w \in \mathbb{C}$. Write $w = a + ib$, where $b \neq 0$.

Then $\exists \ \lambda_0, \lambda_1, \lambda_2, \ldots, \lambda_p > 0$ such that $\lambda_0 + \lambda_1 w + \lambda_2 w^2 + \cdots + \lambda_p w^p = 0$.

PROOF

$z > 0 \Rightarrow Re(z^n) > 0 \quad \forall n$.

(Trivial)

Converse

Supp. $Re(z^n) > 0 \quad \forall n$.

Claim z is a +ve real no.

PROOF

$$\sum_{n=0}^{\infty} \frac{\cos(n\theta)}{2^n} = \frac{4 - 2\cos\theta}{5 - 4\cos\theta}$$

We will use this equation.

WLOG, $|z| = 1$.

$\Rightarrow z = \cos\theta + i\sin\theta = e^{i\theta}$

$R.P.(z^n) > 0 \Rightarrow \cos n\theta > 0 \quad \forall n$ — (1)

If $\theta = 0$, then $z > 0$.

Supp $\theta \neq 0$

is (1) possible?

Look at RHS

$\leq 4/5$

But look at LHS

it is > 1.

Solution to (2)

Solve in the simplest possible case.

$(z = a + ib \ ; \ b \neq 0)$.

Case 1 $a = 0$

Now $z = ib$

How to find $c_0, c_1, \ldots, c_p > 0$ such that $c_0 + c_1 z + c_2 z^2 + \cdots + c_p z^p = 0$?

Case 2 : $Re(z^{n_0}) \leq 0$.

Let Re.part $(z^{n_0}) < 0$. $\to$ (1)

($\exists$ n_0 like this).

Let 'n_0' be the least +ve integer satisfying (1).

$\Rightarrow R.P(z), R.P(z^2), \ldots, R.P(z^{n_0 - 1}) > 0$.

$w = 1 + z + z^2 + \cdots + z^{n_0 - 1} + c z^{n_0} = ki$

for some $c > 0$.

So, $c_0 + c_1 w + \cdots + c_p w^p = 0$ for some $c_0, c_1, \ldots, c_p > 0$.

$\Rightarrow$ we have $c_1', c_2', \ldots, c_q' > 0$

s.t $c_1' + c_2' w + c_2' w^2 + \cdots + c_q' w^q = 0$.

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